Wang et al. *Advances in Difference Equations* (2016) 2016:29
DOI 10.1186/s13662-016-0749-y

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

PD feedback H_∞ control for uncertain singular neutral systems

Jichun Wang[1,2], Qingling Zhang[1,3*] and Dong Xiao[4]

*Correspondence: qlzhang@mail.neu.edu.cn
[1]Institute of Systems Science, Northeastern University, Shenyang, Liaoning 110819, China
[3]State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang, Liaoning 110819, China
Full list of author information is available at the end of the article

Abstract

This paper focuses on the problem of proportional-plus-derivative (PD) feedback H_∞ control for uncertain singular neutral systems. The parameter uncertainties are assumed to be time-varying norm-bounded and appearing not only in the state matrix but also in the derivative matrix. This paper introduces new effective criteria which make the systems stable and meet the H_∞ performance by PD feedback controller. Different from most existing methods, this study attempts to introduce the information between derivative matrices. Based on such an idea, the PD feedback controller for singular neutral systems is first of all proposed which makes the derivative matrices (especially neutral matrix) meet the requirement which guarantees the existence of the solution for the system. So the criteria in this paper are less conservative to some extent. Finally, illustrate examples are given to demonstrate the effectiveness of the proposed approach.

Keywords: H_∞ control; singular neutral system; LMI; PD feedback controller

1 Introduction

Since the late 1980s, interests have been focused on the study of the H_∞ control problem [1] due to its practical and theoretical importance. Various approaches have been developed and a great number of results for continuous systems as well as discrete systems have been reported in the literature: [2] discussed the State space solutions to standard H_2 and H_∞ control problems; [3] studied delay-dependent robust H_∞ controller synthesis for discrete singular delay systems; [4] stated H_∞ control for descriptor systems: a matrix inequalities approach; [5] addressed non-fragile H_∞ control for linear systems with multiplicative controller gain variations. It should be pointed out that all of the works mentioned above are concerned with the H_∞ control problem for conventional state-space systems as well as conventional singular systems. However, many practical processes can be modeled as neutral delay systems [6–8] such as networks of interconnected systems [9], lossless transmission lines, partial element equivalent circuits in electrical engineering, controlled constrained manipulators in mechanical engineering [10, 11], and certain implementation schemes of predictive controllers [12]. So there are many results for the study of the neutral systems [13, 14] performance. For example, [15] studied estimates of perturbation of nonlinear indirect interval control system of neutral type; [16] studied a stabilization method in neutral type direct control systems; [17] studied stabilization of neutral-type indirect control systems to absolute stability state; *etc.* Most of the literature studies the stability [18–20] for the linear neutral systems. But it appears that little results are available so far on H_∞ control of the neutral systems [21, 22] as well as the neutral singular systems [23–25]. The above results are all for the theoretical research of this kind of system. For the practical application research is rarely. In the future, we can achieve more practical results from the application background; for example, results as regards a dissolving tank of chemical process, a large circuit system, and recent general data-driven methods [26, 27].

Circuit analysis, power systems, chemical process simulation, *etc.* can be modeled as a general form of neutral type system, *i.e.* neutral singular systems. Therefore, the study of stability problems for neutral singular systems is of theoretical and practical importance. It is well known that the stability of the differential operator $\Im$ is the prerequisite condition for the stability of neutral systems [28]. Its stability is determined by the derivative matrices. In most previous literature [29–31] the differential operator $\Im$ is presumed to be stable. That is to say, derivative matrices need to meet certain conditions. This has great significance. However, not all of the neutral systems have a stable operator. To make the system meaningful, one should design the controller which makes the operator of the system is stable. Owing to the operator being related to the state with derivative, we only use derivative feedback control to stabilize the operator. To the best of our knowledge, little attention has been given to the neutral singular systems with the operator $\Im$ unstable. Up to now, the proportional-plus-derivative (PD) feedback control problem of neutral singular systems with the operator $\Im$ unstable has not been investigated when the H_∞ performance of the closed-loop system is required. The main objective of this paper is to present the PD feedback controller for neutral singular systems. It can make the systems asymptotically stable with an H_∞ disturbance attenuation $\gamma > 0$ for all nonzero $w(t) \in L_2^P[0, +\infty]$.

The remaining sections of this paper are organized as follows. The neutral singular system description and some relevant lemmas are given in Section 2. Section 3 presents an H_∞ performance analysis of the neutral singular systems and Section 4 designs PD state feedback stabilizing controllers. In Section 5, numerical examples are presented to illustrate the effectiveness of the proposed theoretical results in this paper. Finally, conclusions are given in Section 6.

The following notation will be used throughout the paper: R denotes the set of real numbers. R^n denotes the n-dimensional Euclidean space. $R^{n\times m}$ denotes the set of $n \times m$ matrices with real elements and $R^{n\times m}(s)$ denotes the set of $n \times m$ matrices of rational functions. Let P be a square matrix. The matrix P is symmetric if $P = P^T$. For symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The superscripts 'T' and '$*$' represent the transpose and the complex conjugate transpose. $\wp_{n,\tau} = \wp([-\tau, 0], R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into R^n with the topology of uniform convergence. $\|x\|$ is the Euclidean norm of the vector x. Matrices, if not explicitly stated, are assumed to have compatible dimensions. $\lambda(G, E) = \{\alpha \,|\, \det(G - \alpha E) = 0\}$ means generalized eigenvalue set of matric G and E. $\Omega(0, 1)$ is a circle with 0 as a center and 1 as circle radius.

2 System description and preliminaries

In this paper, we consider the following uncertain singular neutral delay system:

$$(E + \Delta E)\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - h) + (G + \Delta G)\dot{x}(t - \tau) + Bw(t) + B_1 u(t), \tag{1a}$$

$$z(t) = Cx(t). \tag{1b}$$

We have the initial condition

$$x(t_0 + \theta) = \phi(\theta), \quad \forall\theta \in [-\bar{\tau}, 0], (t_0, \phi) \in R^+ \times \wp^{\nu}_{n,\bar{\tau}},$$

where $x(t) \in R^n$ is the state variable vector; $u(t) \in R^m$ is the control input vector; $z(t) \in R^q$ is the output vector; $w(t) \in L_2[0, \infty)$ is the disturbance input vector; and $\phi(t)$ are continuous functions defined on $(-\infty, 0]$. E ($\operatorname{rank}(E) \leq n$), A, A_1, G, B, B_1, C, D are given constant matrices with appropriate dimensions, and $\bar{\tau} = \max\{h, \tau\}$ is a constant time-delay. ΔE, ΔG, ΔA, ΔA_1 are unknown real norm-bounded matrix functions which represent time-varying parameter uncertainties. In this paper, the uncertainties are assumed to be of the form

$$\begin{bmatrix} \Delta E & \Delta G & \Delta A & \Delta A_1 \end{bmatrix} = HF(t)\begin{bmatrix} N_e & N_g & N_a & N_{a1} \end{bmatrix}.$$

Here H, N_a, N_{a1}, N_e, N_g are known real constant matrices with appropriate dimensions, for $\forall t$, $F(t)$ is an unknown real matrix satisfying $F(t)F^T(t) \leq I$. I is a unit matrix with appropriate dimensions.

The nominal unforced singular neutral system of the system (1a) and (1b) can be written as

$$E\dot{x}(t) = Ax(t) + A_1 x(t - h) + G\dot{x}(t - \tau) + Bw(t) + B_1 u(t), \tag{2a}$$

$$z(t) = Cx(t), \tag{2b}$$

$$x(t_0 + \theta) = \phi(\theta), \quad \forall\theta \in [-\bar{\tau}, 0], (t_0, \phi) \in R^+ \times \wp^{\nu}_{n,\bar{\tau}}.$$

In the following, we introduce some of the data that will be used later.

When $E = I$, the system (1a) reduces to the uncertain neutral system with time delays.

Define the operator $\Im : \wp_{n,\tau} \to R^n$ as follows:

$$\Im x_t = Ex(t) - Gx(t - \tau),$$

which will play a major role in the subsequent analysis.

For a given scalar $\gamma > 0$, the H_∞ performance index of the system (2a) and (2b) is defined to be

$$J(w) = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right] dt. \tag{3}$$

Definition 1 [28] Suppose $\Im : C \to R^n$ is linear, continuous and let $C_\Im = \{\Phi \in C : \Im\Phi = 0\}$.

The operator $\Im$ is said to be stable if the zero solution of the homogeneous difference equation

$$\Im y_t = 0, \quad t \geq 0, y_0 = \Psi \in C_{\Im}$$

is uniformly asymptotically stable.

It is noted that the regularity of the neutral singular system (2a) and the stability of operator $\Im$ can ensure the existence and uniqueness of the solution, which is shown in the following lemma.

Lemma 1 *The differential operator $\Im x_t = Ex(t) - Gx(t-\tau)$ is stable if and only if $\lambda(G,E) \subset \{\Omega(0,1) - 0\}$.*

Proof Take the Laplace transform for the equation

$$Ex(t) - Gx(t-\tau) = 0. \tag{4}$$

Then we have

$$\left(E - e^{-s\tau}G\right)X(s) = 0. \tag{5}$$

The characteristic equation of equation (4) is

$$\det\left(E - e^{-s\tau}G\right) = 0.$$

That is,

$$\det\left(-e^{-s\tau}\right)\det\left(-e^{s\tau}E + G\right) = 0.$$

Obviously $\det(-e^{-s\tau}) \neq 0$, so

$$\det\left(G - e^{s\tau}E\right) = 0. \tag{6}$$

Let $e^{s\tau} = \alpha$, then $s = \frac{1}{\tau}\{\ln|\alpha| + i\operatorname{Arg}\alpha\}$.

Equation (6) can be rewritten as

$$\det(G - \alpha E) = 0.$$

If $0 < |\alpha| < 1$, that is, $\{\lambda(G,E) - \{0\}\} \subset \Omega(0,1)$, then $\operatorname{Re} s < 0$.

Conversely, if $\operatorname{Re} s < 0$, then $0 < |\alpha| < 1$, that is, $\{\lambda(G,E) - \{0\}\} \subset \Omega(0,1)$.

The Taylor series expansion of $e^{-s\tau}$ around $s = 0$ is

$$e^{-s\tau} = 1 + (-s\tau) + \frac{1}{2!}(-s\tau)^2 + \frac{1}{3!}(-s\tau)^3 + \cdots. \tag{7}$$

Substituting (7) into (5), we obtain

$$EX(s) - \left[1 + (-s\tau) + \frac{1}{2!}(-s\tau)^2 + \frac{1}{3!}(-s\tau)^3 + \cdots\right]GX(s) = 0. \tag{8}$$

When $s \to 0$, the limit of equation (8) is

$$\lim_{s\to 0}\left\{EX(s) - \left[1 + (-s\tau) + \frac{1}{2!}(-s\tau)^2 + \frac{1}{3!}(-s\tau)^3 + \cdots\right]GX(s)\right\} = 0. \tag{9}$$

According to the final value theorem for the Laplace transform, when $\{\lambda(G,E) - \{0\}\} \subset \Omega(0,1)$, that is to say, all the singularities of $sX(s)$ are in the left half of the s plane, then $\lim_{t\to+\infty} x(t) = \lim_{s\to 0} sX(s)$ is true. Therefore the solution $x(t)$ of equation (4) is asymptotically stable if and only if $\lim_{t\to+\infty} x(t) = \lim_{s\to 0} sX(s) = 0$. Then, equation (9) has the following form:

$$(E - G)X(0) = 0.$$

So $X(0)$ exists and has a unique value if and only if $\det(E - G) \neq 0$. Here $\lambda(G,E) \subset \{\Omega(0,1) - 0\}$ implies $\det(E - G) \neq 0$. Then we have the desired result immediately. This completes the proof. □

Lemma 1 is from the paper 'Robust stability analysis and stabilization of uncertain neutral singular systems', which is accepted by the IJSS.

From Lemma 1, we see that $E - G$ is an invertible matrix. So the system (2a) can be rewritten as the following equation:

$$\dot{x}(t) = \sigma Ax(t) + \sigma A_1x(t-h) + \sigma G\int_{t-\tau}^{t} \ddot{x}(\theta)\,d\theta + \sigma Bw(t) + \sigma B_1u(t), \tag{10}$$

where $\sigma = (E - G)^{-1}$.

It is well known that the stability of the differential operator $\Im$ is the prerequisite condition for the stability analysis of neutral systems. Then according to Lemma 1, derivative matrix needs to meet certain conditions. However, only the derivative feedback changes the derivative matrix. So the purpose of this paper is to design a derivative feedback controller which makes the system (1a) and (1b) asymptotically stable with an H_∞ disturbance attenuation $\gamma > 0$ for all nonzero $w(t) \in L_2^p[0,+\infty]$.

Lemma 2 *For any vectors a and b of appropriate dimensions*, $X = X^T > 0$ *satisfying* $X^TX < I$, *we have*

$$-2a^Tb < a^TX^{-1}a + b^TX^TXb.$$

Lemma 2 is from the paper 'Robust stability analysis and stabilization of uncertain neutral singular systems', which is accepted by the IJSS.

Lemma 3 [32] *Let A*, *L*, *E*, *and F be real matrices of appropriate dimensions*, *with F satisfying* $F^TF \le I$. *Then we have*:

i. *For any scalar* $\varepsilon > 0$,

$$LFE + E^TF^TL^T \le \varepsilon^{-1}LL^T + \varepsilon E^TE.$$

ii. *For any matrix* $P > 0$ *and scalar* $\varepsilon > 0$ *such that* $\varepsilon I - EFE^T > 0$,

$$(A + LFE)^T P(A + LFE) \leq A^T PA + A^T PE(\varepsilon I - E^T PE)^{-1} E^T PA + \varepsilon L^T L.$$

3 H_∞ performance analysis

In this section, we will focus on the H_∞ performance analysis of the neutral singular systems (1a), (1b) and (2a), (2b) and establish the following results.

Theorem 1 *For given scalar* $\tau, h > 0$, *the nominal system* (2a) *and* (2b) *with* $u(t) = 0$ *is asymptotically stable with an* H_∞ *disturbance attenuation* $\gamma > 0$ *for all nonzero* $w(t) \in L_2^p[0, +\infty]$ *if there exist positive matrices* $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2)$, $U = U^T > 0$, *and any matrices* N_j $(j = 1, 2)$, *such that the following LMIs are feasible*:

$$(E - G)P(E - G)^T > 0, \tag{11}$$

$$\begin{bmatrix} -I & U \\ U^T & -I \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \Upsilon_1 & \Upsilon_2 & \Upsilon_3 & 0 & 0 & B & \Upsilon_4 & \Upsilon_5 & 0 & 0 \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_6 & N_1 & 0 & 0 & 0 & \Upsilon_8 & 0 \\ * & * & * & * & \Upsilon_7 & 0 & 0 & 0 & 0 & U \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -Q_1 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & -U & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{13}$$

where

$$\Upsilon_1 = (E - G)PA^T + AP(E - G)^T, \qquad \Upsilon_2 = A_1 Q_1,$$
$$\Upsilon_3 = -G, \qquad \Upsilon_4 = (E - G)PC^T, \qquad \Upsilon_5 = (E - G)P,$$
$$\Upsilon_6 = \tau^2 Q_2 - N_1 C - C^T N_1^T, \qquad \Upsilon_7 = N_2^T + N_2, \qquad \Upsilon_8 = C^T N_2^T.$$

Proof Due to $E - G$ is an invertible matrix, we have $(E - G)P(E - G)^T > 0$.

Next, we choose a candidate Lyapunov functional $V(x(t))$ as follows:

$$V(x(t)) = V_1 + V_2 + V_3,$$
$$V_1 = x^T(t)P^{-1}x(t),$$
$$V_2 = \int_{t-h}^{t} x^T(\alpha)Q_1^{-1}x(\alpha)\, d\alpha,$$
$$V_3 = \tau \int_{-\tau}^{0} \int_{t+\beta}^{t} \ddot{x}^T(\theta)Q_2\ddot{x}(\theta)\, d\theta\, d\beta.$$

For any matrices N_1, N_2 with appropriate dimensions, we obtain

$$2\bigl[\ddot{x}^T(t)N_1 + \ddot{z}^T(t)N_2\bigr]\bigl[\ddot{z}(t) - C\ddot{x}(t)\bigr] = 0.$$

Calculating the derivative of $V(x(t))$ along the solution of the nominal system (10) yields

$$\dot{V}\bigl(x(t)\bigr) + Z^T(t)Z(t) - \gamma^2 w^T(t)w(t) \le \begin{bmatrix} x(t) \\ x(t-h) \\ \int_{t-\tau}^{t} \ddot{x}(\theta)\,d\theta \\ \ddot{x}(t) \\ \ddot{y}(t) \\ w(t) \end{bmatrix}^T \Omega \begin{bmatrix} x(t) \\ x(t-h) \\ \int_{t-\tau}^{t} \ddot{x}(\theta)\,d\theta \\ \ddot{x}(t) \\ \ddot{y}(t) \\ w(t) \end{bmatrix}, \tag{14}$$

$$\Omega = \begin{bmatrix} A^T\chi^T + \chi A + Q_1^{-1} + C^TC & \chi A_1 & -\chi G & 0 & 0 & \chi B \\ * & -Q_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & \tau^2 Q_2 - N_1C - C^TN_1^T & N_1 - C^TN_2^T & 0 \\ * & * & * & * & N_2 + N_2^T & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

where $\chi = P^{-1}(E-G)^{-1}$, $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2)$.

From (12), (13), the Schur complement lemma, and Lemma 2, we have $\Omega < 0$. That is, $\dot{V}(x_t) + Z^T(t)Z(t) - \gamma^2 w^T(t)w(t) < 0$. The initial condition $x(0) = 0$ implies that $J(w) < 0$. This completes the proof. □

Remark 1 Most researchers solve the differential operator stability problems by the use of derivative matrices with decomposition [19]. That is, there exist two invertible matrices K and S such that

$$KES = \begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$KGS = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}.$$

This requires the matrices E and G to be very special. It is not generally true to make two matrices diagonalization simultaneously. Furthermore, the corresponding computational complexities are large. However, this theorem holds as long as $\det(E-G) \neq 0$. So Theorem 1 is more effective than the previous results.

Theorem 2 *For given scalar $\tau, h > 0$, the uncertain neutral system* (1a) *and* (1b) *is asymptotically stable with an H_∞ disturbance attenuation $\gamma > 0$ for all nonzero $w(t) \in L_2^P[0, +\infty]$ if there exist positive matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2)$, $U_i = U_i^T > 0$ $(i = 1, 2)$ and any matrices N_j $(j = 1, 2)$ and scalar $\varepsilon_j > 0$, $j = 1, \ldots, 6$, such that the following LMIs are feasible*:

$$\begin{bmatrix} (G-E)P(E-G)^T + \varepsilon_6 HH^T & (E-G)P(N_e - N_g)^T \\ (N_e - N_g)P(E-G) & -\varepsilon_6 I \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} -I & U_i \\ U_i^T & -I \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix}
\Upsilon_{11} & \Upsilon_2 & \Upsilon_3 & 0 & 0 & B & \Upsilon_4 & \Upsilon_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H & 0 \\
* & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{12} & 0 & 0 & 0 & 0 \\
* & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{13} \\
* & * & * & \Upsilon_6 & N_1 & 0 & 0 & 0 & \Upsilon_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Upsilon_7 & 0 & 0 & 0 & 0 & U_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{14} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{15} & 0 & 0 \\
* & * & * & * & * & * & * & * & -U_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & \Upsilon_{16} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_5 I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -U_2 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix}$$

$$< 0, \tag{17}$$

where

$$\Upsilon_{11} = \Upsilon_1 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5)HH^T, \qquad \Upsilon_{12} = Q_1^T N_{a1}^T, \qquad \Upsilon_{13} = N_g^T U_1^T,$$

$$\Upsilon_{14} = CP(N_e - N_g)^T, \qquad \Upsilon_{15} = P(N_e - N_g)^T, \qquad \Upsilon_{16} = N_a P(N_e - N_g)^T.$$

Proof Let

$$\mathrm{A} = (G - E)P(E - G)^T + \varepsilon_6 HH^T + \varepsilon_6^{-1}(E - G)P(N_e - N_g)^T(N_e - N_g)P(E - G). \tag{18}$$

According to the Schur complement lemma, (15) is equivalent to $\mathrm{A} < 0$.

From (18) and Lemma 3, we obtain

$$\begin{aligned}
\mathrm{A} &\geq (G - E)P(E - G)^T + (G - E)P(N_e - N_g)^T F^T(t)H^T \\
&\quad + HF(t)(N_g - N_e)P(E - G)^T \\
&> (G - E)P(E - G)^T + (G - E)P(\Delta E - \Delta G)^T \\
&\quad + (\Delta G - \Delta E)P(E - G)^T - (\Delta E - \Delta G)P(\Delta E - \Delta G)^T.
\end{aligned}$$

That is,

$$(E + \Delta E - G - \Delta G)P(E + \Delta E - G - \Delta G)^T > 0.$$

If E, G, A and A_1 in (13) are replaced by $E + HF(t)N_e$, $G + HF(t)N_g$, $A + HF(t)N_a$, and $A_1 + HF(t)N_{a1}$, respectively, then (13) is rewritten as the following formula:

$$\begin{aligned}
&\Phi + \Gamma_h F(t)\Gamma_{neg}^T + \Gamma_{neg}F^T(t)\Gamma_h^T + \Gamma_h F(t)\Gamma_{na1}^T + \Gamma_{na1}F^T(t)\Gamma_h^T \\
&\quad + \Gamma_h F(t)\Gamma_{negc}^T + \Gamma_{negc}F^T(t)\Gamma_h^T + \Gamma_h F(t)\Gamma_{ne}^T + \Gamma_{ne}F^T(t)\Gamma_h^T \\
&< 0,
\end{aligned}$$

where

$$\Phi = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 & \Upsilon_3 & 0 & 0 & B & \Upsilon_4 & \Upsilon_5 & 0 & 0 \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_6 & N_1 & 0 & 0 & 0 & \Upsilon_8 & 0 \\ * & * & * & * & \Upsilon_7 & 0 & 0 & 0 & 0 & U \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -Q_1 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & -U & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\Gamma_h^T = \begin{bmatrix} H^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{neg} = \begin{bmatrix} (N_e - N_g)PN_a^T F^T(t)H^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{na1} = \begin{bmatrix} 0 & N_{a1}Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{negc} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & (N_e - N_g)PC^T & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{ne} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & (N_e - N_g)P & 0 & 0 \end{bmatrix}.$$

By Lemma 3, for the system (1a) and (1b) there exists $\varepsilon_i > 0$ ($i = 1, \dots, 6$), applying the Schur complement lemma again, we see that (16) and (17) hold. This completes the proof. □

4 Stabilizing controllers of uncertain neutral singular system

In this section we design PD state feedback controllers to make the resulting closed loop systems asymptotically stable with an H_∞ disturbance attenuation $\gamma > 0$ for all nonzero $w(t) \in L_2^p[0, +\infty]$.

For the system (1a) and (1b), our interest is to design a state derivative feedback controller

$$u(t) = k_e\dot{x}(t) + k_g\dot{x}(t - \tau) + k_a x(t) + k_{a1}x(t - h). \tag{19}$$

When we apply the controller (19) to the system (1a), the resulting closed loop system can be written as

$$E_c\dot{x}(t) = A_c x(t) + A_{1c}x(t - \tau) + G_c\dot{x}(t - \tau) + Bw(t), \tag{20}$$

where

$$E_c = E + \Delta E - B_1 k_e,$$

$$G_c = G + \Delta G + B_1 k_g,$$

$$A_c = A + \Delta A + B_1 k_a,$$

$$A_{1c} = A_1 + \Delta A_1 + B_1 k_{a1}.$$

The following results present the solvability conditions for the state feedback stabilizing control problems for the uncertain neutral singular system (1a) with the controller (19).

Theorem 3 *For given scalar $\tau, h > 0$, considering the neutral singular system* (2a) *and* (2b), *there exists a controller* (19) *such that the resulting closed loop system* (20) *without parameter uncertainties is asymptotically stable with an H_∞ disturbance attenuation $\gamma > 0$ for all nonzero $w(t) \in L_2^P[0, +\infty]$ if there exist positive matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2)$, $U = U^T > 0$ and any matrices N_j $(j = 1, 2)$, X, X_a X_{a1}, K_g such that the following LMIs are feasible*:

$$\begin{bmatrix} \mathrm{M} & 0 & 0 & -B_1X \\ 0 & -P & -X^TB_1^T & 0 \\ 0 & -B_1X & \mathrm{M} & 0 \\ -X^TB_1^T & 0 & 0 & -P \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} -I & U \\ U^T & -I \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \Upsilon_1 & \Upsilon_2 & \Upsilon_3 & 0 & 0 & B & \Upsilon_4 & \Upsilon_5 & 0 & 0 & B_1X & B_1X_a \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_6 & N_1 & 0 & 0 & 0 & \Upsilon_8 & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_7 & 0 & 0 & 0 & 0 & U & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -U & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -P \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned} &\mathrm{M} = (G - E)P(E - G)^T + (G - E)X^TB_1^T + B_1X(E - G)^T, \\ &\Upsilon_1 = (E - G)PA^T + AP(E - G)^T + (E - G)X_a^TB_1^T + B_1X_a(E - G)^T - B_1XA^T - AX^TB_1^T, \\ &\Upsilon_2 = A_1Q_1 + B_1X_{a1}, \qquad \Upsilon_3 = -G - B_1K_g, \qquad \Upsilon_4 = (E - G)PC^T - B_1XC^T, \\ &\Upsilon_5 = (E - G)P - B_1X, \qquad \Upsilon_6 = \tau^2Q_2 - N_1C - C^TN_1^T, \qquad \Upsilon_7 = N_2^T + N_2, \Upsilon_8 = C^TN_2^T. \end{aligned}$$

In this case, the PD state H_∞ controller can be chosen as

$$u(t) = \bigl(XP^{-1} - K_g\bigr)\dot{x}(t) + K_g\dot{x}(t - \tau) + X_aP^{-1}x(t) + X_{a1}Q_1^{-1}x(t - h). \tag{24}$$

Proof In (11), E is replaced by $E + B_1K_e$, then (11) is rewritten as the following condition:

$$(E - B_1K_e - G - B_1K_g)P(E - B_1K_e - G - B_1K_g)^T > 0. \tag{25}$$

Let $K = K_e + K_g$, $X = KP$.

According to the Schur complement lemma, (25) becomes the following LMI:

$$\begin{bmatrix} M & jB_1X \\ jX^TB_1^T & -P \end{bmatrix} < 0, \tag{26}$$

where j is the imaginary unit. Then (26) is equivalent to the LMI (21).

If E, G, A, and A_1 in (13) are replaced by $E + B_1K_e$, $G + B_1K_g$, $A + B_1K_a$, and $A_1 + B_1K_{a1}$, respectively, let $X_a = K_aP$, $X_{a1} = K_{a1}Q_1$, according to the Schur complement lemma and Lemma 3, (13) is equivalent to the LMI (23). This completes the proof. □

Remark 2 As far as we know the methods of controller design for neutral systems do not include a derivative feedback because of the complexity of the neutral term in neutral systems. Theorem 3 provides the derivative feedback method which uses the knowledge of complex matrix inequalities [33]. So the gain matrices are all real matrices.

Remark 3 Based on this theorem, there is no longer the assumption that the differential operator $\Im$ is to be stable. For example, for the nominal system (2a) with the coefficient matrices $E = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $G = \left[\begin{smallmatrix} 0.2 & 0 \\ 0.2 & 10 \end{smallmatrix}\right]$, according the previous conclusions, see [17], we will not be able to judge the performance of the system. This is so because most results are based on the assumption that the differential operator $\Im$ is stable. But through Theorem 3, we can make the differential operator $\Im$ stable and then design the controller.

Theorem 4 *For given scalar* $\tau, h > 0$, *considering the uncertain neutral singular system* (1a) *and* (1b), *there exists a controller* (19) *such that the resulting closed loop system* (20) *is asymptotically stable with an* H_∞ *disturbance attenuation* $\gamma > 0$ *for all nonzero* $w(t) \in L_2^p[0, +\infty]$ *if there exist positive matrices* $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2)$, $U_i = U_i^T > 0$ $(i = 1, 2)$, *and any matrices* N_j $(j = 1, 2)$, X, X_a, X_{a1}, K_g *and scalar* $\varepsilon_j > 0, j = 1, \ldots, 6$ *such that the following LMIs are feasible*:

$$\begin{bmatrix} \psi_1 & -\Psi_2 \\ -\Psi_2 & \psi_1 \end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} -I & U_i \\ U_i^T & -I \end{bmatrix} < 0, \tag{28}$$

$$\left[\begin{array}{c|c} \Omega & \begin{array}{c} \Omega_1 \\ 0_{16\times 2} \end{array} \\ \hline * & \Omega_2 \end{array}\right] < 0, \tag{29}$$

where

$$\mathrm{M}_1 = (G - E)P(E - G)^T + (E - G)X^TB_1^T + B_1X(E - G)^T + \varepsilon_6HH^T,$$

$$\mathrm{M}_2 = (E - G)P(N_e - N_g)^T - B_1X(N_e - N_g)^T,$$

$$\begin{aligned} \mathrm{T}_{11} = {} & (E - G)PA^T + AP(E - G)^T + (E - G)X_a^TB_1^T + B_1X_a(E - G)^T \\ & - B_1XA^T - AX^TB_1^T + \sum_{i=1}^{5} \varepsilon_iHH^T, \end{aligned}$$

$$\Psi_1 = \begin{bmatrix} M_1 & M_2 & 0 \\ \ast & -\varepsilon_6 & 0 \\ \ast & \ast & -P \end{bmatrix}, \qquad \Psi_2 = \begin{bmatrix} 0 & 0 & B_1X \\ \ast & 0 & 0 \\ \ast & \ast & 0 \end{bmatrix},$$

$$\Omega_1 = \begin{bmatrix} B_1X & B_1X_a \end{bmatrix}, \qquad \Omega_2 = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix},$$

$$\Omega = \begin{bmatrix}
\mathrm{T}_{11} & \mathrm{T}_2 & \mathrm{T}_3 & 0 & 0 & B & \mathrm{T}_4 & \mathrm{T}_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H & 0 \\
\ast & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{12} & 0 & 0 & 0 & 0 \\
\ast & \ast & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{13} \\
\ast & \ast & \ast & \Upsilon_6 & N_1 & 0 & 0 & 0 & \Upsilon_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \Upsilon_7 & 0 & 0 & 0 & 0 & U_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & -I & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{14} & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{15} & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -U_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_1 I & \Upsilon_{16} & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_3 I & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_4 I & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_5 I & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -U_2 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -I
\end{bmatrix}$$

$$< 0.$$

Proof The proof can be carried out by following a similar line to the proof of Theorem 3, and thus it is omitted. □

Remark 4 Obviously, the normal condition $\det(E - G) \neq 0$ guarantees the existence and uniqueness of solutions to the neutral singular systems. So knowing the information of derivative matrices is particularly important. The main improvement in the theorems in this paper is that the derivative matrices E and G can become the desired matrices by the PD feedback controller. Generally speaking, a static state feedback cannot get the desired results.

5 Numerical examples

In this section, we provide numerical examples to show the effectiveness and applicability of the results proposed in this paper.

Example 1 Consider the nominal system (2a). The parameters of the system are assumed as follows:

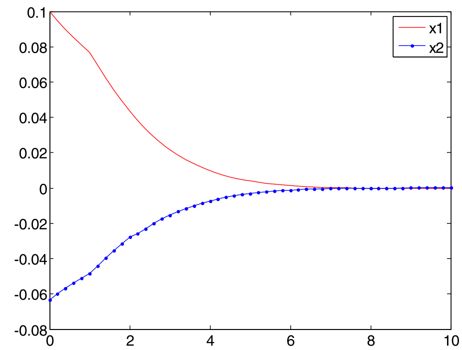
$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} -0.1 & 0 \\ 10 & 1 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.1 & 2 \\ 2 & -1 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -0.1 & 0.2 \\ 0 & -3 \end{bmatrix}, \qquad B = 0, \qquad C = I.$$

Obviously in this system $\det(E - G) = -0.1 \neq 0$ satisfies the condition (11) of Theorem 1, so the solution of the system exists. In terms of Theorem 1, when $\tau \leq 0.0868$, this system is asymptotically stable.

With the initial conditions $x(t) = [0.1 \ \ -0.062]^T$, the numerical simulation is carried out using the Simulink in Matlab. The result of the numerical simulation is presented in Figure 1, which shows the asymptotical stability of the system.

Figure 1 **State response of nominal system.**



Example 2 Consider the nominal system (2a) and (2b) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 0 \\ 0 & -0.9 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}, \qquad B = 0, \qquad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad C = I,$$

when $u(t) = 0$, by solving LMIs conditions in Theorem 1 via the Matlab LMI toolbox, we cannot obtain feasible solutions. That is, the nominal neutral singular system is not stable. The state responses of the system with initial conditions $x(t) = [0.1\ 0]^T$ are given in Figure 2(a). Solving LMIs in Theorem 3 with $\gamma = 0.1$ via the Matlab LMI toolbox, we can get the feasible solutions:

$$P^{-1} = \begin{bmatrix} 180.4680 & 18.0523 \\ 18.0523 & 1.8059 \end{bmatrix}, \qquad Q_1^{-1} = \begin{bmatrix} 25.4010 & -6.3496 \\ -6.3496 & 1.5875 \end{bmatrix}.$$

This solution leads to the following state feedback controller gains:

$$K_e = \begin{bmatrix} -3.8181 & -0.4296 \end{bmatrix}, \qquad K_g = \begin{bmatrix} 0 & -0.0525 \end{bmatrix},$$
$$K_a = \begin{bmatrix} 31.3809 & 4.7320 \end{bmatrix}, \qquad K_{a1} = \begin{bmatrix} 0.9409 & 0.7651 \end{bmatrix},$$

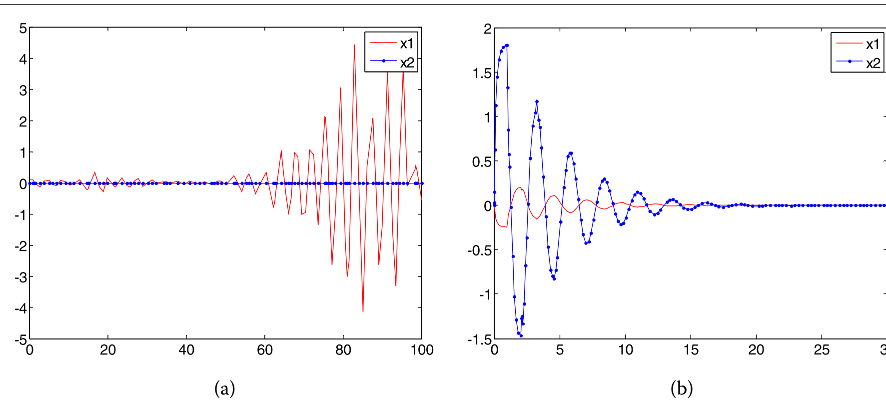


Figure 2 **The state response of Example 2 when (a)** $u(t) = 0$**, (b)** $u(t) \neq 0$**.**

and the PD state feedback controller is

$$u(t) = (K - K_g)\dot{x}(t) + K_g\dot{x}(t-\tau) + K_a x(t) + K_{a1}x(t-h).$$

The state responses of the closed-loop system with the initial condition $x(t) = [0.1\ 0]^T$ is given in Figure 2(b), which demonstrates the applicability of the proposed method.

Example 3 Consider the uncertain neutral singular system (1a) and (1b). The parameters of the system are assumed as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} -0.8 & 0 \\ -0.2 & -0.8 \end{bmatrix}, \qquad A = \begin{bmatrix} -2 & 0 \\ 0 & -15 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad C = I,$$
$$H = 1, \qquad N_a = N_{a1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \qquad N_e = N_g = 0.$$

It is assumed that the uncertain matrix is given as $F(t) = [\sin t\ \ \sin t]^T$.

We take $\gamma = 0.1$, when $u(t) = 0$, by solving the LMIs conditions in Theorem 2 via the Matlab LMI toolbox, we cannot obtain feasible solutions. That is, the uncertain neutral singular system is not stable. The state responses of the system with the initial conditions $x(t) = [0.1\ 0.1]^T$ are given in Figure 3(a). By solving LMIs in Theorem 4 via the Matlab LMI toolbox, we can get the feasible solutions:

$$P^{-1} = \begin{bmatrix} 0.1287 & -0.0644 \\ -0.0644 & 0.0322 \end{bmatrix}, \qquad Q_1^{-1} = \begin{bmatrix} 7.3028 & 3.6372 \\ 3.6372 & 1.8169 \end{bmatrix}.$$

This solution leads to the following PD state feedback controller gains:

$$K_e = \begin{bmatrix} 0.9788 & -0.3651 \end{bmatrix}, \qquad K_g = \begin{bmatrix} 0.3491 & 0.6012 \end{bmatrix},$$
$$K_a = \begin{bmatrix} -8.3221 & 3.2812 \end{bmatrix}, \qquad K_{a1} = \begin{bmatrix} 1.8420 & -1.5787 \end{bmatrix}.$$

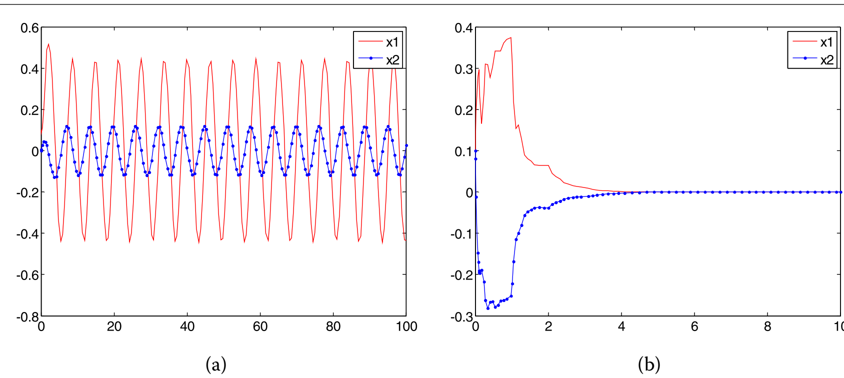


Figure 3 The state response of Example 3 when (a) $u(t) = 0$, (b) $u(t) \neq 0$.

It is assumed that the uncertain matrix E_c, G_c in closed-loop system (20) are given as $F(t) = [\sin t\ \ \sin t]^T$. For any $t \in [0, +\infty]$ and with the above designed controller, the determinant of the derivative matrices of the corresponding closed-loop system is

$$\det(E_c - G_c) = 0.3977 \neq 0,$$

which implies that the derivative matrix $E_c - G_c$ of the closed loop system is invertible.

The state responses of the closed-loop system with initial condition $x(t) = [0.1\ 0.1]^T$ is given in Figure 3(b), which demonstrates the applicability of the proposed method.

6 Conclusions

In this article, we have dealt with the H_∞ performance analysis and the H_∞ control problem for uncertain neutral singular systems with norm-bounded parameter uncertainties. By using the free-weight matrix method and the LMI technique, we have novel criteria which ensure the neutral singular system under consideration to be stable while satisfying a prescribed H_∞ performance level. Based on this, sufficient conditions for the existence of PD state H_∞ controllers have been proposed. Compared with some existing results, the obtained PD state H_∞ controller can change the derivative matrices. Based on the criteria, numerical examples have been provided to illustrate the effectiveness of the proposed method.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors read and approved the final manuscript.

Author details
[1]Institute of Systems Science, Northeastern University, Shenyang, Liaoning 110819, China. [2]College of Science, Liaoning University of Technology, Jinzhou, Liaoning 121001, China. [3]State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang, Liaoning 110819, China. [4]School of Information Science and Engineering, Northeastern University, Shenyang, Liaoning 110819, China.

Acknowledgements
The authors would like to thank the anonymous reviewers for their helpful and constructive comments to improve the quality of this note. This paper is supported by the National Natural Science Foundation of China (No. 61203214 and No. 61273008).

Received: 5 August 2015 Accepted: 10 January 2016 Published online: 27 January 2016

References

1. Ren, J-C, Zhang, Q-L: Non-fragile PD state H_∞ control for a class of uncertain descriptor systems. Appl. Math. Comput. **218**, 8806-8815 (2012)
2. Doyle, JC, Glover, K, Khargonekar, PP, Francis, BA: State space solutions to standard H_2 and H_∞ control problems. IEEE Trans. Autom. Control **34**(8), 831-847 (1989)
3. Feng, Z, Lam, J, Gao, H: Delay-dependent robust H_∞ controller synthesis for discrete singular delay systems. Int. J. Robust Nonlinear Control **21**(16), 1880-1902 (2011)
4. Masubuchi, I, Kamitane, Y, Ohara, A, Suda, N: H_∞ control for descriptor systems: a matrix inequalities approach. Automatica **33**(4), 669-673 (1997)
5. Yang, G-H, Wang, J-L: Non-fragile H_∞ control for linear systems with multiplicative controller gain variations. Automatica **37**(5), 727-737 (2001)
6. Chen, Y, Zheng, W-X, Xue, A: A new result on stability analysis for stochastic neutral systems. Automatica **46**(12), 2100-2104 (2010)
7. He, Y, Wu, M: Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. Syst. Control Lett. **51**(1), 57-65 (2004)
8. Fridman, E: New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems. Syst. Control Lett. **43**(4), 309-319 (2001)
9. Hu, H-L, Zhao, D, Zhang, Q-L: Observer-based decentralized control for uncertain interconnected systems of neutral type. Math. Probl. Eng. **2013**, Article ID 134935 (2013). doi:10.1155/2013/134935

10. Hu, G-D, Liu, M-Z: Stability criteria of linear neutral systems with multiple delays. IEEE Trans. Autom. Control **52**(4), 720-724 (2007)
11. Muthukumar, P, Rajivganthi, C: Approximate controllability of impulsive neutral stochastic functional differential system with state-dependent delay in Hilbert spaces. J. Control Theory Appl. **11**(3), 351-358 (2013). doi:10.1007/s11768-013-2061-7
12. Niculescu, SI, Brogliato, B: Force measurements time-delays and contact instability phenomenon. Eur. J. Control **5**, 279-289 (1999)
13. Kuang, J-X, Tian, H-J, Shan, K-T: Asymptotic stability of neutral differential systems with many delays. Appl. Math. Comput. **217**, 10087-10094 (2011)
14. Baštinec, J, Diblík, J, Khusainov, DY, Ryvolová, A: Exponential stability and estimation of solutions of linear differential systems of neutral type with constant coefficients. Bound. Value Probl. (2010). doi:10.1155/2010/956121
15. Shatyrko, AV, Khusainov, DY, Diblík, J, Baštinec, J, Ryvolová, A: Estimates of perturbation of nonlinear indirect interval control system of neutral type. J. Autom. Inf. Sci. **43**(1), 13-28 (2011). doi:10.1615/JAutomatInfScien.v43.i1.20
16. Shatyrko, AV, Khusainov, DY: On a stabilization method in neutral type direct control systems. J. Autom. Inf. Sci. **45**(12), 1-10 (2013). doi:10.1615/JAutomatInfScien.v45.i12.10
17. Shatyrko, A, Nooijen, RRP, Kolechkina, A, Khusainov, DY: Stabilization of neutral-type indirect control systems to absolute stability state. Adv. Differ. Equ. (2015). doi:10.1186/s13662-015-0405-y
18. Zhao, Y-X, Ma, Y-C: Stability of neutral-type descriptor systems with multiple time-varying delays. Adv. Differ. Equ. **2012**, 15 (2012). doi:10.1186/1687-1847-2012-15
19. Li, H, Li, H-B, Zhong, S-M: Stability of neutral type descriptor system with mixed delays. Chaos Solitons Fractals **33**(5), 1796-1800 (2007)
20. Chen, W-H, Zheng, W-X: Delay-dependent robust stabilization for uncertain neutral systems with distributed delays. Automatica **43**(1), 95-104 (2007)
21. Xu, S-Y, Lam, J, Zhou, Y, Sun, M-H: H_∞ control for neutral systems via non-fragile state feedback controller. In: International Conference on Control, Automation, Robotics and Vision, Kunming, China, 6-9 December, pp. 48-51 (2004). http://hdl.handle.net/10722/46614
22. Xia, H-W, Zhao, P-P: A novel approach to H_∞ control design for linear neutral time-delay systems. Math. Probl. Eng. **2013**, Article ID 526017 (2013). doi:10.1155/2013/526017
23. Mahmoud, MS: Robust H_∞ control of linear neutral systems. Automatica **36**, 757-764 (2000). doi:10.1016/S0005-1098(99)00159-4
24. Deng, S-J, Liao, X-F, Guo, S-T: Asymptotic stability analysis of certain neutral differential equations: a descriptor system approach. Math. Comput. Simul. **79**(10), 2981-2993 (2009)
25. Lan, Q-X, Feng, C-Y: State feedback stabilization of neutral type descriptor systems. In: 2009 World Congress on Computer Science and Information Engineering, pp. 247-251 (2009). doi:10.1109/CSIE.2009.125
26. Yin, S, Zhu, X, Kaynak, O: Improved PLS focused on key performance indicator related fault diagnosis. IEEE Trans. Ind. Electron. **62**(3), 1651-1658 (2014)
27. Yin, S, Zhu, X: Intelligent particle filter and its application on fault detection of nonlinear system. IEEE Trans. Ind. Electron. **62**(6), 3852-3861 (2015). doi:10.1109/TIE.2015.2399396
28. Hale, JK: Theory of Functional Differential Equations. Springer, New York (1977). doi:10.1007/978-94-015-8084-7
29. Li, X-G, Zhu, X-J: Stability analysis of neutral systems with distributed delays. Automatica **44**(8), 2197-2201 (2008)
30. Liu, X-G, Wu, M, Martin, R, Tang, M-L: Stability analysis for neutral systems with mixed delays. J. Comput. Appl. Math. **202**(2), 478-497 (2007)
31. Wang, H-J, Xue, A: Delay-dependent robust stability criteria for neutral singular systems with time-varying delays and nonlinear perturbations. In: American Control Conference, pp. 5446-5451 (2009). doi:10.1109/ACC.2009.5160318
32. de Souza, CE, Li, X: Delay-dependent robust H_∞ control of uncertain linear state-delayed systems. Automatica **35**(7), 1313-1321 (1999)
33. Li, Y: Robust Control - Linear Matrix Inequality Approach. Tsinghua University Press, Beijing (2002). http://www.docin.com/p-638565754.html